# General Disclaimer

## One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

NASA Contractor Report 174872

Translational and Extensional Energy Release Rates (the J- and M-Integrals)
for a Crack Layer in Thermoelasticity

A. Chudnovsky and B. Gommerstadt

Case Western Reserve University
Cleveland, Ohio

March 1985

# TRANSLATIONAL AND EXTENSIONAL ENERGY RELEASE RATES

## (THE J- AND M-INTEGRALS) FOR A CRACK LAYER IN THERMOELASTICITY

A. Chudnovsky and B. Gommerstadt
Department of Civil Engineering
Case Western Reserve University
Cleveland, Ohio 44106

## I. INTRODUCTION

Since 1979 there has been a number of papers on evaluation of energy release rate for thermoelasticity and corresponding J-integral. Two main approaches are developed to treat energy release rate in elasticity. The first is based on direct calculation of potential energy rate with respect to crack length [1]. The second makes use of Lagrangian formalism (Nether's theorem) [2]. The direct method was extended to the problems of thermoelasticity by Wilson and Yu [3], K. Kishimoto, S. Aoki, and M. Sakata [4] and Gurtin [5]. A useful discussion on the extension of the approach to thermoelasticity was presented by McCartney [6].

The second approach was elaborated by Atkinson and Smelser [7] and discussed in [8]. In elasticity the energy release rate is expressed as a path independent integral. The path invariancy of the similar integrals for thermoelasticity is discussed in aforementioned papers.

In [3, 4] the J-integral consist of two parts: first is a common for elastic problems integral along a path and the second includes a nonvanishing integral over the volume (area for two dimensional applications).

Path independent integral introduced in [5] has limitations in path invariancy as was noticed in [7]: the integration paths should start and end at certain points. It is not discussed in [5] whether the integral represents any energy release rate.

Another path independent integral expressed in terms of energy-momentum tensor has been derived in [7]. The derivation follows the Lagrangian formalism of [2] and essentially depends on Lagrangian chosen. A comparison of this integral and that obtained in present paper (see also [6]) indicates that an entropy term is omitted.

In this paper we consider translational and expansional energy release rates employing the formalism of irreversible thermodynamics and Crack Layer Approach. A damage zone usually precedes and surrounds a propagating crack. A system of a crack and the associated damage is treated as a crack layer (CL) [9]. The damage parameter for simplicity is defined as an area of discontinuity surfaces per unit volume. A part of the CL within which the rate of damage accumulation is positive, $(\dot{\rho} > o)$ is defined as an active zone. The CL theory thus models fracture propagation as the active zone movements: translation and rotation as a rigid, isotropic expansion and distortion for a small active zone. There are potential energy release rates corresponding to each of these elementary movements. Energy release rates corresponding only to the active zone translation, the J-integral, and expansion, the M-integral are considered here. The rate of the damage density resulting from these two movements of active zone is [10,11].

$$\dot{\rho} = -V_1 \partial_1 \rho - \dot{e} X_\kappa \partial_\kappa \rho$$

(1)

where $v_1$ and $\dot{e}$ stands for the rates of translation in direction tangent to the crack path and expansion, respectively, and X is a position vector with respect to the center of isotropic expansion.

Potential energy release rate P associated with damage growth is

$$\dot{P} = \int_V \frac{\partial \gamma}{\partial \rho} \dot{\rho} \, dv$$

(2)

2

where $\pi$ is a potential energy density and integral is taken over the active zone. Upon substitution of (1) this relation takes the form

$$\dot{P} = -v_i \int_V \frac{\partial \pi}{\partial \rho} \partial_i \rho \, dv - \dot{e} \int_V \frac{\partial \pi}{\partial \rho} x_k \partial_k \rho \, dv \qquad (3)$$

where k = 1,2

The first integral in the right-hand side of (3) is a translational energy release rate (J-integral), the second one is an expansional energy release rate (M-integral).

2. Translational Energy Release Rate in Thermoelasticity

The stress tensor $\underset{\sim}{\sigma}$ and the absolute temperature T constitutes a conventional set of parameters for a thermoelastic medium. This set, extended by adding the damage parameter $\rho$, is chosen as a system of thermodynamic state parameters

$$\{\underset{\sim}{\sigma}, T, \rho\}$$

The corresponding thermodynamic potential is a Gibbs' free energy density which is usually designated in continuum mechanics as the potential energy density

$$\pi = \pi(\sigma, T, \rho) \qquad (4)$$

Then, the following constitutive equations for thermoelasticity are held:

$$\left. \frac{\partial \pi}{\partial T} \right|_{\sigma = CONST} = -S \qquad (5)$$

$$\left. \frac{\partial \pi}{\partial \underset{\sim}{\sigma}} \right|_{T = CONST} = -\underset{\sim}{\varepsilon} \qquad (6)$$

3

where S is the entropy, and $\varepsilon$ is the total strain tensor. According to Eq. (3).

$$J_1 = - \int_V \frac{\partial \pi}{\partial \rho} \, \partial_1 \rho \, dv \tag{7}$$

Since (4)

$$\frac{\partial \pi}{\partial \rho} \partial_k \rho = \partial_k \pi - \frac{\partial \pi}{\partial \sigma_{ij}} \partial_k \sigma_{ij} - \frac{\partial \pi}{\partial T} \partial_k T \tag{8}$$

Using local constitutive equations (5) and (6) this expression can be rewritten as

$$\frac{\partial \pi}{\partial \rho} \partial_k \rho = \partial_k f - \partial_j (\sigma_{ij} u_{i,k}) + S \partial_k T \quad (i,j,k = 1,2) \tag{9}$$

Here, f is the strain energy density, or the specific Helmholtz free energy, and $u_i$ the component of the displacement vector.

For $\alpha$ being a linear coefficient of thermal expansion the specific entropy for thermoelasticity

$$s = \alpha \, \sigma_{kk} \tag{10}$$

Substituting (9) and (10) into expression (7) for $J_1$ and using Gauss theorem we finally arrive at the integral

$$J_1 = \int_\Gamma [f n_1 - \sigma_{ij} n_j u_{i,1}] d\Gamma + \int_V \alpha \, \sigma_{kk} \, \theta_{,1} \, dv \tag{11}$$

4

Here, $n_j$ is j-th component of the unit normal to an integration path $\Gamma$, $\theta = T - T_0$, where $T_0$ is the absolute temperature for the initial equilibrium state of the system when the strain is everywhere zero. The first term in (11), i.e., integral along the path $\Gamma$ repeats that for elastic case with the fields $f$, $\underset{\sim}{\sigma}$ and $\underset{\sim}{u}$ corresponding to the thermoelastic equilibrium. The second term, i.e., the area integral, cannot be in general converted into a line integral. However, such a conversion can be easily done for linearly varying temperature field.

3. Expansional Energy Release Rate

Expression for the M-integral follows from the second part of the right-hand side of (3):

$$M = \int_V \frac{\partial \pi}{\partial \rho} X_k \, \partial_k \rho \, dV \tag{12}$$

Using the same approach like in the previous section the expression for this integral is obtained in the following form.

$$M = \oint_\Gamma [X_k \eta_k f - \sigma_{ij} \eta_j X_k u_{i,k}] d\Gamma + \int_V \alpha \sigma_{\ell\ell} (\theta + X_k \theta_{,k}) dV \tag{13}$$

where i, j, k and $\ell = 1,2$.

The position vector $X_k$ is taken with respect to the center of isotropic expansion.

The M-integral consists of two parts similarly to the J-integral (11): one is a line integral and another is an area integral.

4. Discussion

Since $\rho$ is not zero within the active zone $V_A$ only the following integral over an arbitrary area V $V_A$ is reduced to

5

$$\int_{V V_A} \frac{\partial \pi}{\partial \rho} \dot{\rho} \, dV = \int_{V_A} \frac{\partial \pi}{\partial \rho} \dot{\rho} \, dV \qquad (14)$$

It leads to the following relations

$$\int_{V \cdot_A} \frac{\partial \pi}{\partial \rho} \partial_{,} \rho \, dV = \int_{V_A} \frac{\partial \pi}{\partial \rho} \partial_{,} \rho \, dV \qquad (15)$$

and

$$\int_{V V_A} \frac{\partial \pi}{\partial \rho} X_\kappa \partial_\kappa \rho \, dV = \int_{V_A} \frac{\partial \pi}{\partial \rho} X_\kappa \partial_\kappa \rho \, dV \qquad (16)$$

These imply the invariancy of J- and M-integral with respect to the domain of integration V  V$_A$. As it was shown above the area integral can be decomposed into the linear and area integrals. The linear integral formally coincides with that for the elastic case, however, for thermoelasticity it is not path independent. Only the sum of the linear and area integrals in (11) and (13) obeys the conservation law.

The expressions (11) and (13) agree with corresponding integrals of [12] and differ from all the rest mentioned above expressions for energy release rate.

6

**References**

1. J. R. Rice, Trans., ASME. J. Appl. Mech., 90, 379-386 (1968).

2. Eshelby J. D. in "Inelastic Behavior of Solids" (Ed. M. F. Kanninen et al.) p. 77. McGraw-Hill, New York (1970).

3. Wilson, W. K., and Yu, I. W., Int. J. of Fracture, Vol. 15, No. 4, p. 377-387, (1979).

4. Kishimoto, K., Aoki, S., and Sakata, M., Engineering Fracture Mechanics, Vol. 13, pp. 841-850 (1980).

5. Gurtin, M. E., Int. J. of Fracture, Vol. 15, No. 5, pp. R169-R170 (1979).

6. McCartney, L. M., Int. J. of Fracture, Vol. 15, No. 6, pp. R217-R221 (1979).

7. Atkinson, L. and Smelser, R. E. Int. J. Solids and Structures, Vol. 18, pp. 533-549 (1982).

8. Frankfort, G. and Golebiewska Herman,, A., Trans ASME, J. of Appl. Mech., Vol. 49, p. 710-714 (1982).

9. Chudnovsky, A., NASA Report, N174634, March, 1984.

10. V. Khandogin, A. Chudnovsky, In the book "Dynamics and Strength of Aircraft Constructions", 4 p. 148-175, Novosibirsk, 1978 (in Russian).

11. Chudnovsky, A., Workshop on Damage Eds. Stouffer D. et al. Kentucky, May 4-7, p. 79-87.

12. Aoki, S., Kishimoto, K., Sakata, M., Trans. ASME, J. Appl. Mech., Vol. 48, p. 825-829 (1981).